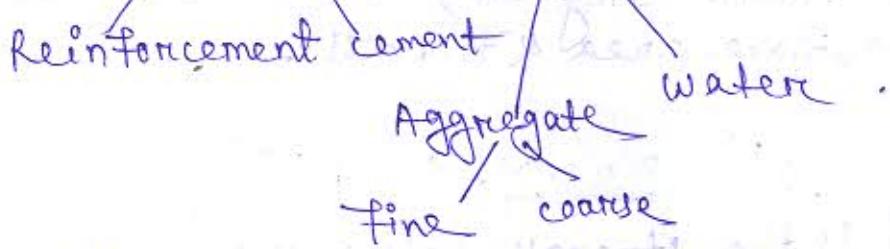
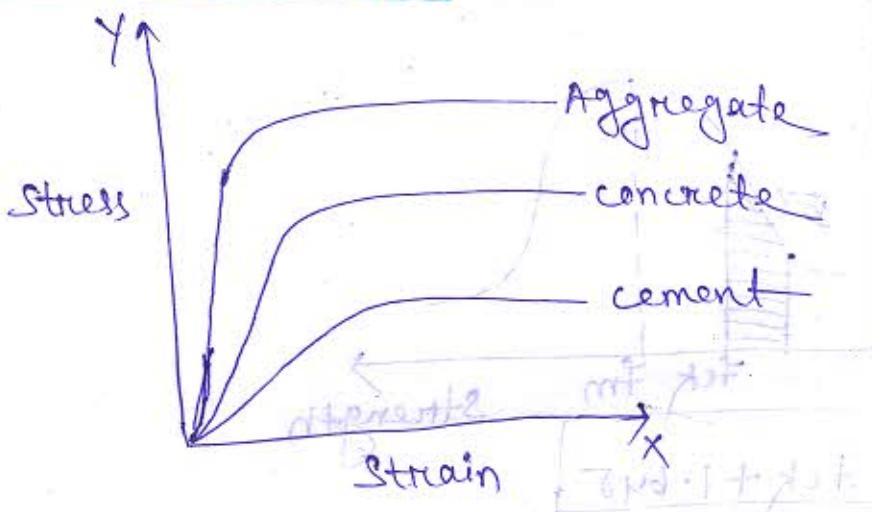


## Reinforced cement concrete:-



If it is the mixture of cement, fine aggregate, coarse aggregate & required quantity of water.

## Stress-strain line:-



## Cement:-

cement was developed by Joseph Aspdin.  
Generally 3 grades of cement are available.

1. C<sub>33</sub>

2. C<sub>43</sub>

3. C<sub>53</sub>

33 represents the compressive strength of cement mortar cube of size 70.7 mm at 28 days.

'C' represents the mixture of cement & sand.

Unit of 33, 43 & 53 is in MPa i.e. 33 N/mm<sup>2</sup>.

There is no difference between the different grades of cement. They only differ as per their specific surface area & fineness.

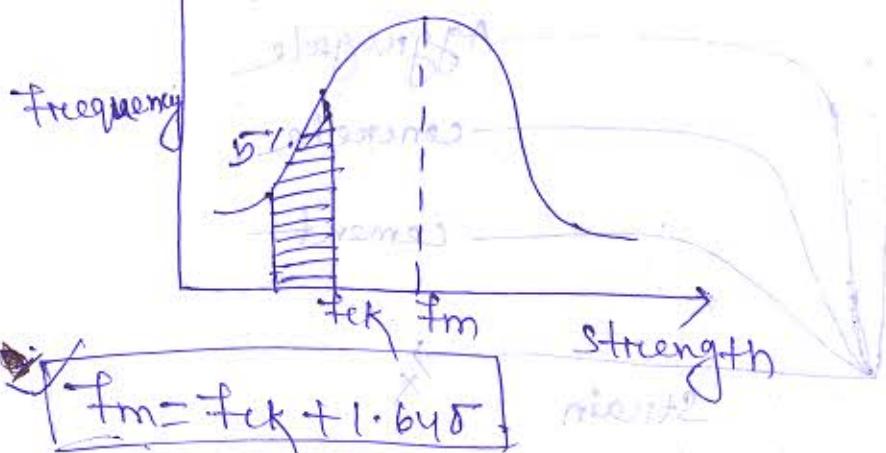
### Characteristics Strength ( $f_{ck}$ ):-

$f_f$  is the strength below which not more than 5% of test result are expected to fall.

If 100 cubes are tested atleast 95 cubes

Should pass the test result.

Relationship



$\sigma$  = Standard deviation

$f_m$  = Mean Strength

When no. of samples ( $n$ ) is  $> 30$ ,

$$\sigma = \sqrt{\frac{\sum (f - f_m)^2}{n-1}}$$

When  $n \leq 30$ ,

$$\sigma = \sqrt{\frac{\sum (f - f_m)^2}{n-1}} \text{ for } n \leq 30$$

On Head  
For  $n \geq 30$   $\sigma$  =  $\frac{\sum (f - f_m)^2}{n-1}$

$$M_{10} - M_{15} \quad 3.5$$

$$M_{20} - M_{25} \quad 4$$

$$M_{30} - M_{35} \quad 5$$

→  $M_{25}$  represent the characteristic strength  
( $f_{ck} = 25 \text{ N/mm}^2$ ) of the mix 'M'.

Q:- Find out the target mean strength of  $M_{35}$ ,  
 $M_{25}$  &  $M_{15}$  concrete.

Ans:- For  $M_{35}$  concrete,

$$\begin{aligned} f_m &= f_{ck} + 1.64\sigma \\ &= 35 + 1.64 \times 5 \\ &= 43.2 \text{ N/mm}^2 \end{aligned}$$

For  $M_{25}$  concrete,

$$\begin{aligned} f_m &= f_{ck} + 1.64\sigma \\ &= 25 + 1.64 \times 4 \\ &= 31.56 \text{ N/mm}^2 \end{aligned}$$

For  $M_{15}$  concrete,

$$\begin{aligned} f_m &= f_{ck} + 1.64\sigma \\ &= 15 + 1.64 \times 3.5 \\ &= 20.74 \text{ N/mm}^2 \end{aligned}$$

### Factors affecting the Strength of concrete:-

Dimension of cylinder =  $150 \times 300 \text{ mm}$ .

Dimension of the cube =  $150 \times 150 \times 150 \text{ mm}$ .

1. Shape:

Cylinder strength =  $0.8 \times$  cube strength  
=  $0.8 \times f_{ck}$ .

Cube strength =  $1.25 \times$  cylinder strength.

## 2. Size:

The 10cm size cube strength is 5% more than that of 15cm cube due to better homogeneity & quality control.

## Water cement ratio:

According to Abrahams law water cement ratio is inversely proportional to the strength of concrete that means if we decrease the water cement ratio then the strength of concrete will be enhanced.

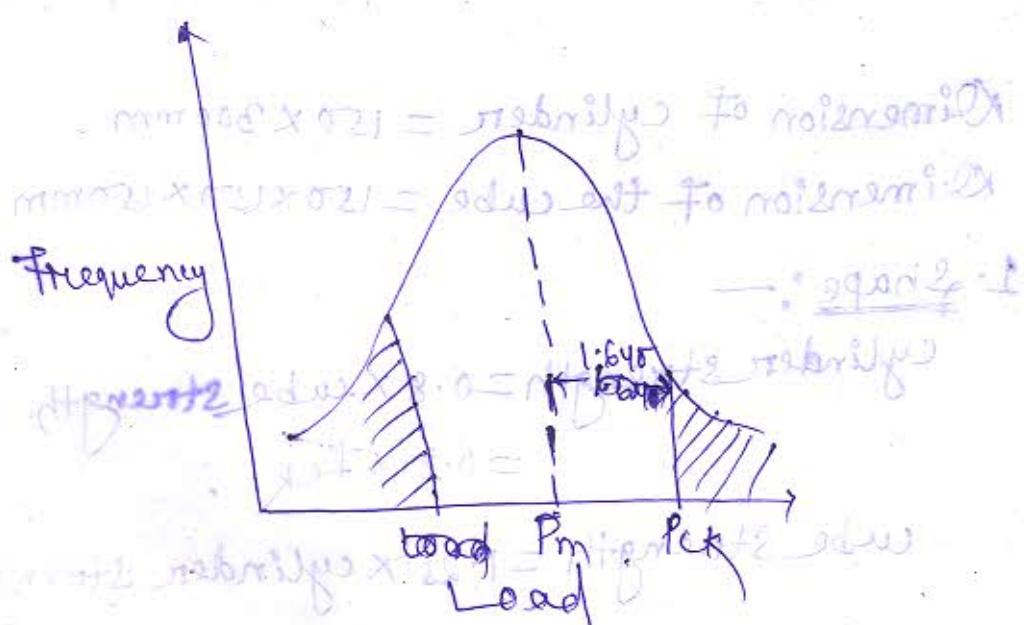
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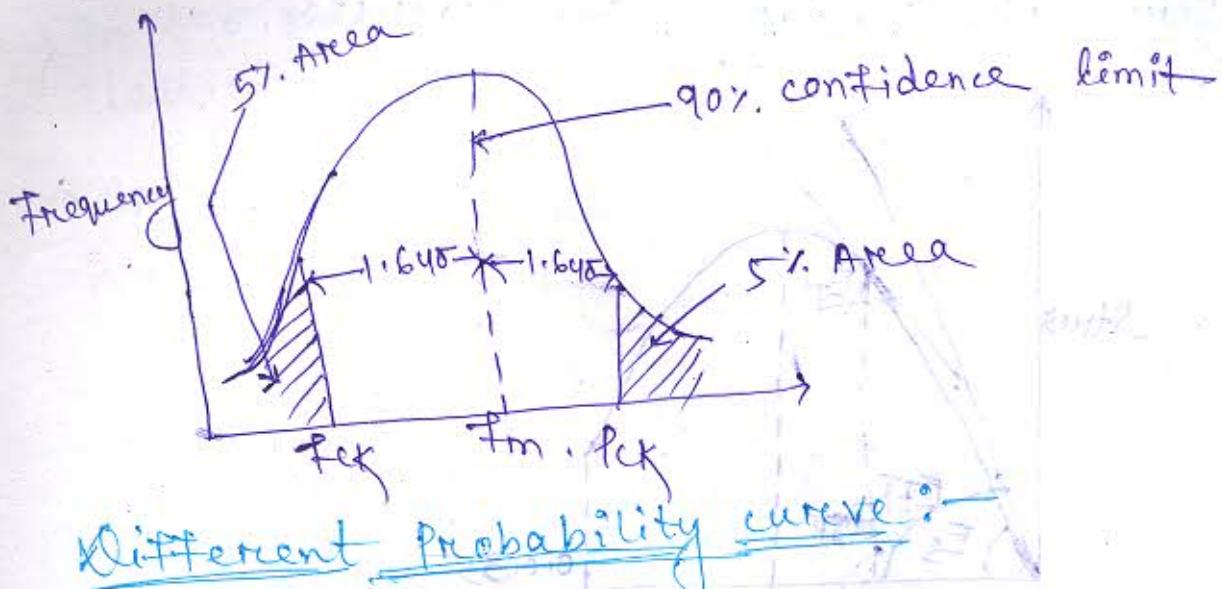
## Characteristics load:

It is the load which is having 95% probability of not being exceeded during entire life of the structure.

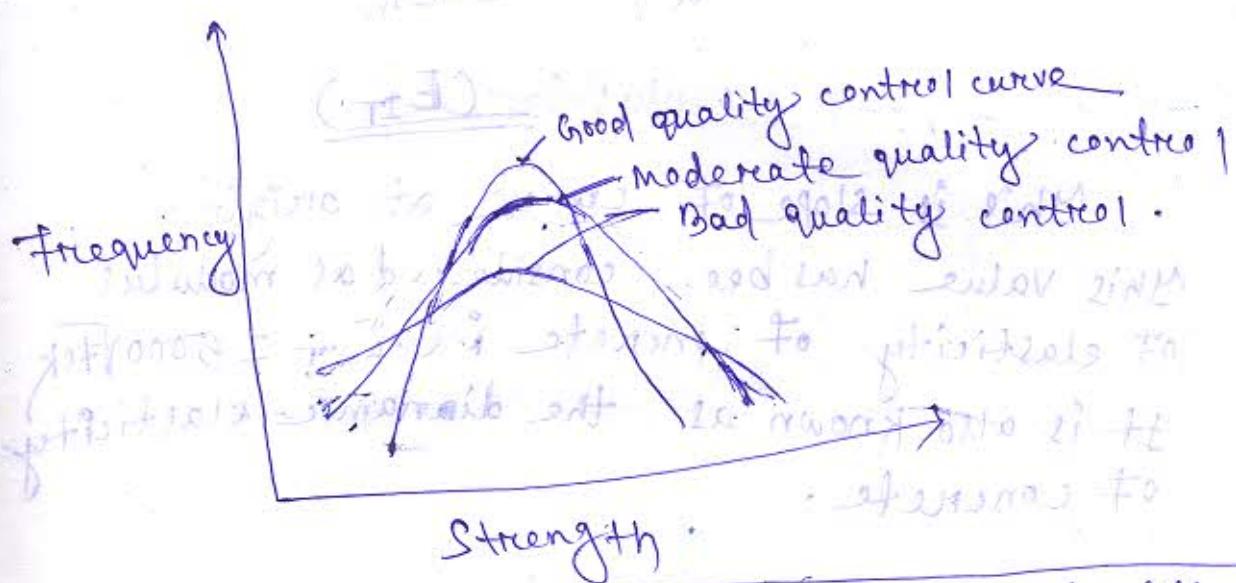
$$f_m = f_{ck} + 1.64\sigma$$

$$\boxed{P_m = P_{ck} + 1.64\sigma}$$





Different Probability curve :-



Design load = Load factor × characteristics strength.

Young's modulus of elasticity of concrete :-

→ It is represented as  $E_c$ .

$$E_c = 5000 \sqrt{f_{ck}}$$

To calculate  $f_{ck}$  takes it values from

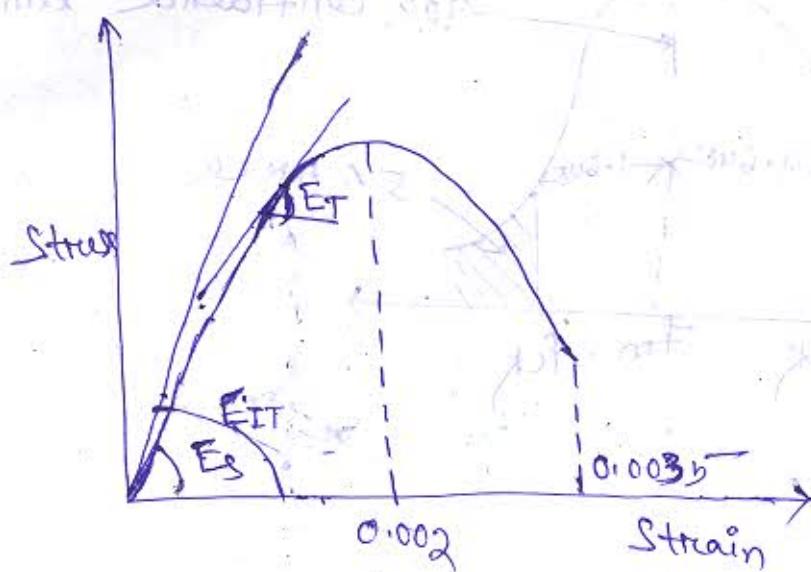
Q:- Find out the Young's modulus of M45 grade concrete?

$$\text{Ans: } E_c = 5000 \sqrt{f_{ck}}$$

$$= 5000 \sqrt{45}$$

$$= 33541.02 \text{ MPa}$$

## Stress-strain diagram for concrete:



→ Initial Tangent modulus :- ( $E_{IT}$ )

This is slope of curve at origin.  
This value has been considered as modulus of elasticity of concrete i.e.  $E_{IT} = 5000 \text{ fck}$ .  
It is also known as the dynamic elasticity of concrete.

→ Second modulus of elasticity ( $E_s$ )

It is the slope of line joining any point on the curve with the origin.

→ Tangent modulus of elasticity ( $E_T$ ) :-

Slope of tangents at any point on the curve is called tangent modulus of elasticity.

$$E_{IT} = 5000 \text{ fck} = 5000 \times 10^6 \text{ N/m}^2$$

$$E_s = 20000 \text{ N/mm}^2$$

- In the linear region all the modulus of elasticity are same i.e.  $E_{IT} = E_s = E_T$ .
- In non-linear region  $E_{IT}$  is greater than  $E_s$  is greater than  $E_T$  ( $E_{IT} > E_s > E_T$ ).

### Effect of creep on $E_c$ :-

1. Time depended deformation (excluding strain due to shrinkage & temperature of total strain).
2. Creep occurs due to dead load only.

$$E_c = \frac{\delta}{\epsilon_e}$$

$$E_{ce} = E_c(1 + \theta t)$$

$$E_{ce} = \frac{E_c}{1 + \theta t}$$

$\theta$  = creep co-efficient

### Tensile Strength of concrete :-

Direct tensile strength of the concrete cannot be measured because it is very difficult to perform the tensile strength test.

→ As the compressive strength increases the tensile strength also increases. But the rate of increase in compressive strength is more than that of increase in tensile strength.

## 2. Flexural strength

of the concrete :-

$$F_{cr} = 0.7 \times f_{ck}$$

Date - 18/01/2019

## Split tensile strength :-

$$\text{split tensile stress} = \frac{2P}{\pi D L}$$

Where, P = Load

D = Diameter

L = Length

## Steel reinforcement

The different types of reinforcement used in reinforced concrete are :-

1. Mild steel,

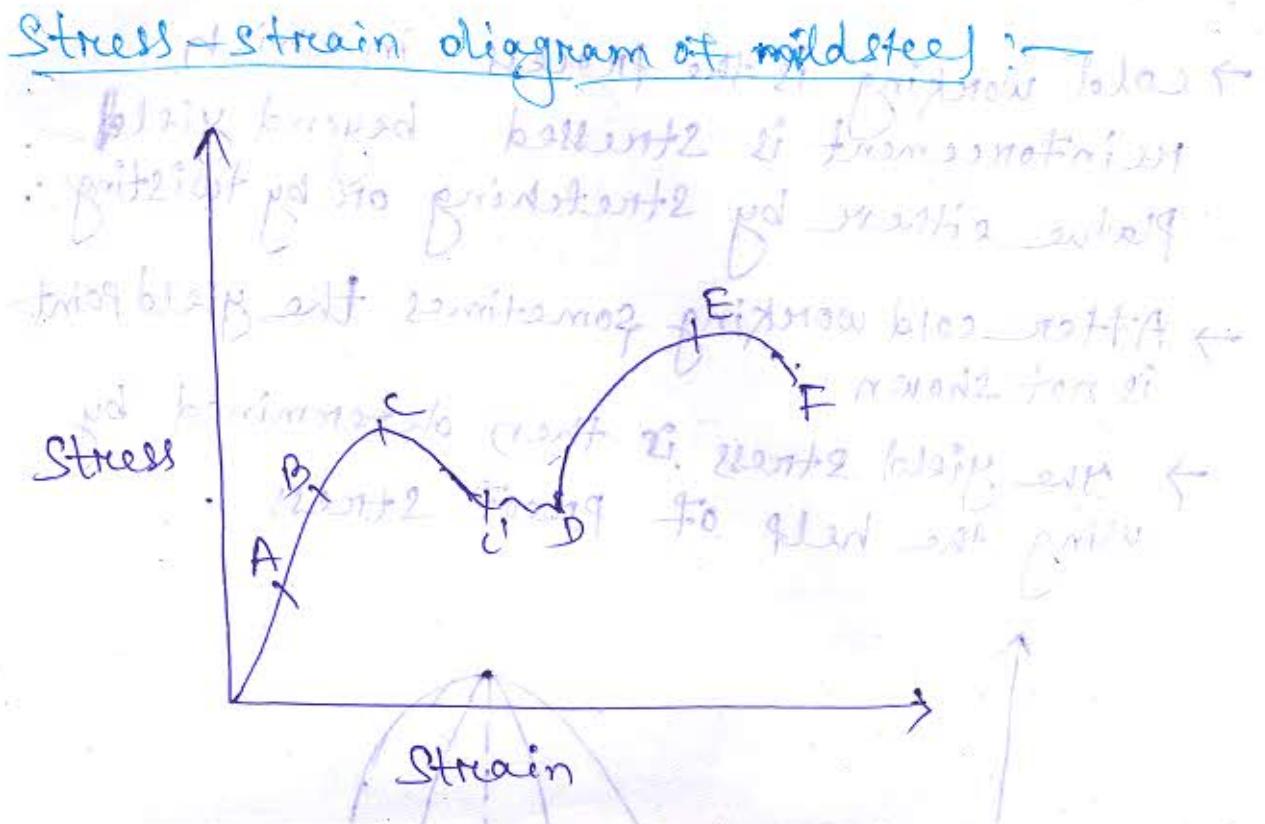
(a) ordinary steel, (b) hot rolled steel.

2. Medium tensile steel.

3. High yield steel deformed bar (HYSI)

4. cold twisted deformed bar (CTD)

5. TMT bar (Thermo mechanically treated bar).



A → Limit of Proportionality

B → Elastic Limit

C → Upper yield Point

D → Lower yield point

E → Ultimate Point

F → Breaking Point

DE → Strain Hardening Zone

EF → Strain Softening Zone

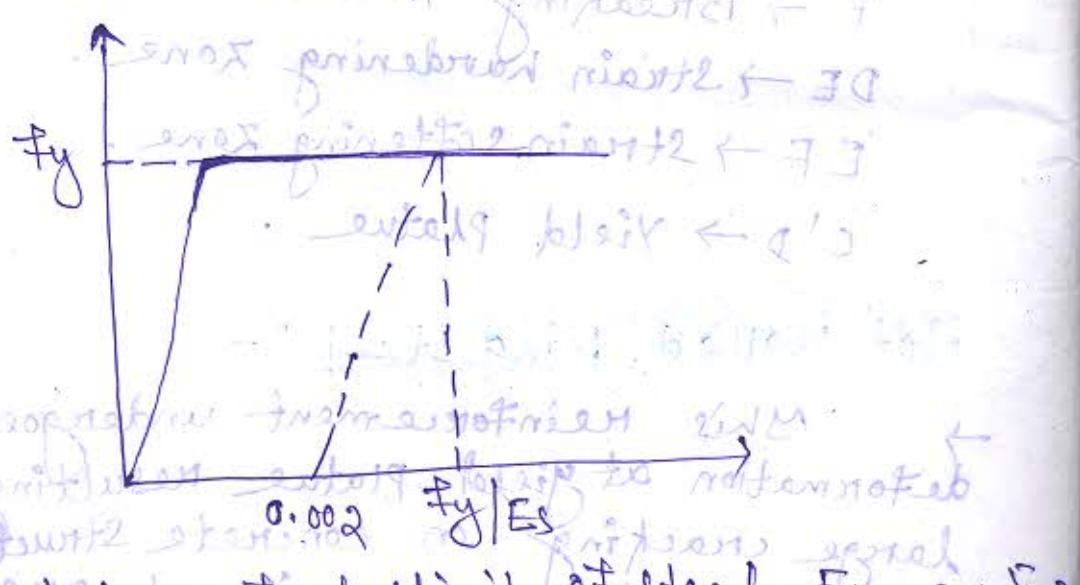
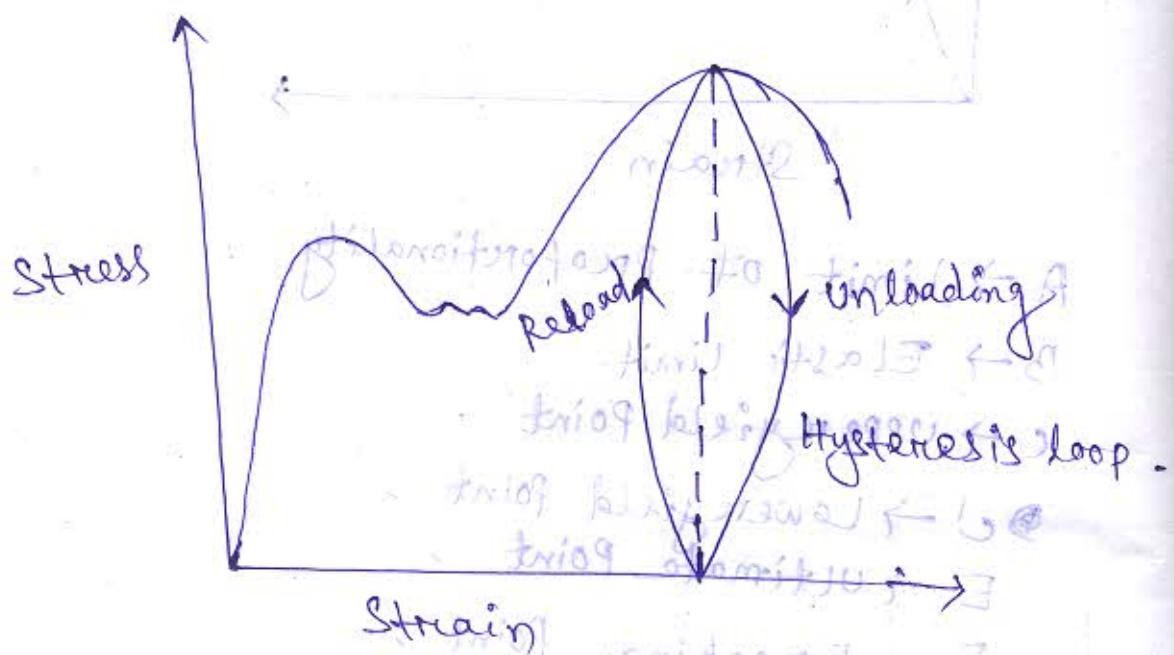
C'D → Yield Plateau

### Hot rolled Mild Steel :-

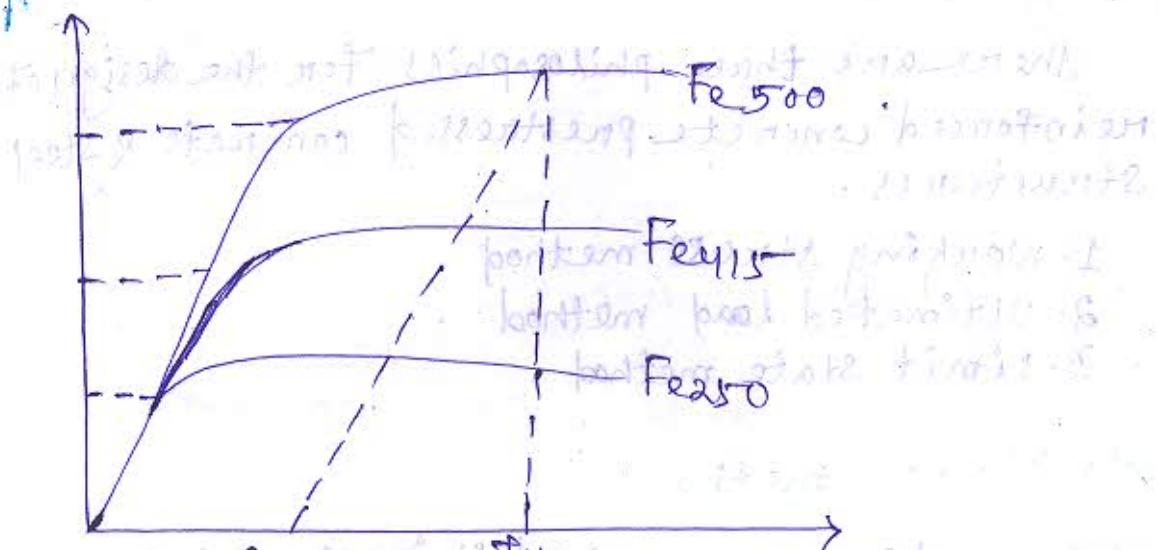
→ This reinforcement undergoes large deformation at yield plateau resulting to large cracking on concrete structure. This type of behaviour is not accepted.

→ Yield plateau can be avoided by cold working.

- cold working is the process in which reinforcement is stressed beyond yield plateau either by stretching or by twisting.
- After cold working sometimes the yield point is not shown.
- the yield stress is then determined by using the help of proof stress. (see 2)



→ Modulus of elasticity of steel,  $E_s = 2 \times 10^5 \text{ MPa}$   
 $E_s = 200 \text{ GPa}$



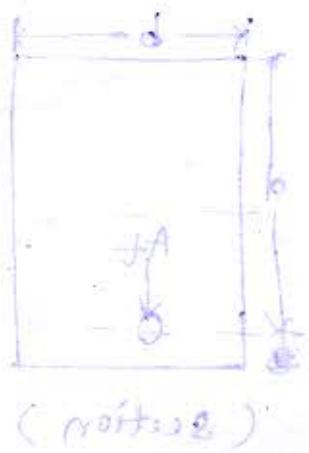
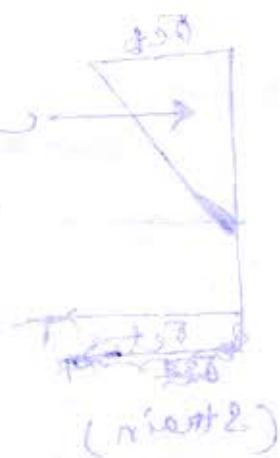
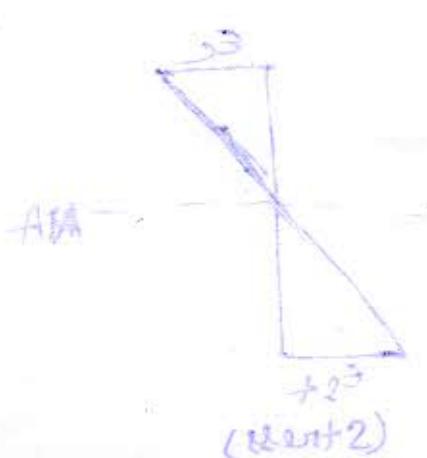
TMT bar is  $\leftarrow$  concrete & steel coated

TMT bar is having both strength & ductility that is why it is mostly used now-a-days. When the cut end of TMT bar are dipped in  $\text{HNO}_3$  solution (nitric acid + methanol) three distinct layers appear.

Outer Martensite, anticorrosive layer.

High strength, martensite + bainite

Quenched core, ferrite, Martensite, anticorrosive layer  
Bainite, perlite matrix



## Design philosophies:-

There are three philosophies for the design of reinforced concrete prestressed concrete & steel structures.

1. Working stress method
2. Ultimate load method
3. Limit state method

### 1. Working stress method :-

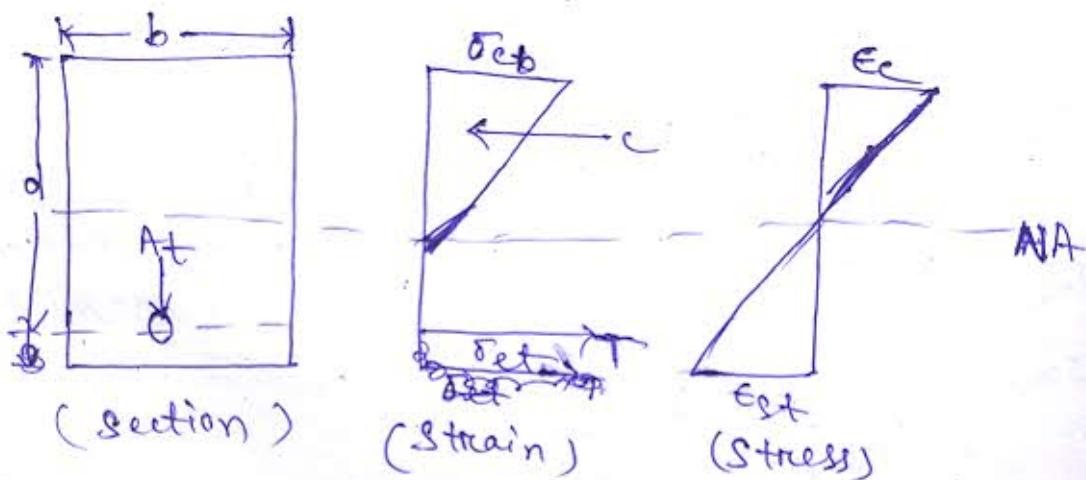
→ In working stress method it is assumed that concrete is elastic, steel & concrete act together elastically & the relationship between loads & stresses is linear right upto the collapse of the structure.

→ The basis of the method is that the permissible stress for concrete & steel are not exceeded anywhere in the structure when it is subjected to the worst combination of working loads.

→ The section is designed in accordance with the elastic theory of bending assuming that both materials obey the Hook's law.

→ The elastic theory assumes a linear

Variation of Strain & Stress from zero at neutral axis to a maximum at the extreme fibre.



- $A_t$  = Area of tension steel  
 $b$  = Width of section  
 $C$  = Total compressive force  
 $D$  = Depth of section  
 $d$  = Effective depth of section, defined as the depth from extreme compressive fibre to center of tensile steel  
 $l_{el}$  = Lever arm, defined as the distance between the point of application of force of compression & force of tension.  
 $N_d$  = Depth of neutral axis  
 $T$  = Total force of tension  
 $\sigma_{cn}$  = Permissible comp. stress in concrete  
 $\sigma_{st}$  = Permissible tensile stress in Steel  
 $E_c$  = Compressive strain in concrete  
 $E_s$  = Tensile strain in Steel.

### Assumptions:

- A section which is plane before bending remains after bending. This is also referred to as Bernoulli's assumption.
- Bond between steel & concrete is perfect within the elastic limit of steel.
- Tensile strength of concrete is ignored.
- Concrete is plastic i.e. the stress in concrete varies linearly from zero at the neutral axis to a maximum at the extreme fibre.
- The modular ratio in has the value  $\frac{E_s}{E_c}$  where  $E_c$  is the permissible comp. stress in bending in N/mm<sup>2</sup> or MPa.

IS:456-2000 uses a factor of safety equal to 3, on the 28 days cube strength to obtain the permissible comp. stress in bending in concrete; & equal to 1.78 on the yield strength of steel in tension to obtain the permissible tensile strength in reinforcement. Thus for properly designed structural elements, the stresses computed under the action of working loads will be well within the elastic range.

Working stress method can be expressed as

$$UR \geq L$$

$$UR = \frac{R}{\text{Factor of safety}} \geq L$$

which is always less than unity.

R - Resistance of the structural element.

L - Working loads on the structural element.

### Drawbacks of working stress method:-

- (i) concrete is not elastic. The inelastic behaviour of concrete starts right from very low stresses.
- (ii) The actual stress distribution in a concrete section can not be described by a triangular stress diagram.
- (iii) Since Factor of safety is on the stresses under working loads, there is no way to account for different degrees of uncertainty associated with different types of loads. With elastic theory it is impossible to determine the actual factor of safety with respect to loads.
- (iv) It is difficult to account for shrinkage & creep effects by using the working stress method.

## 2. Ultimate Load method:-

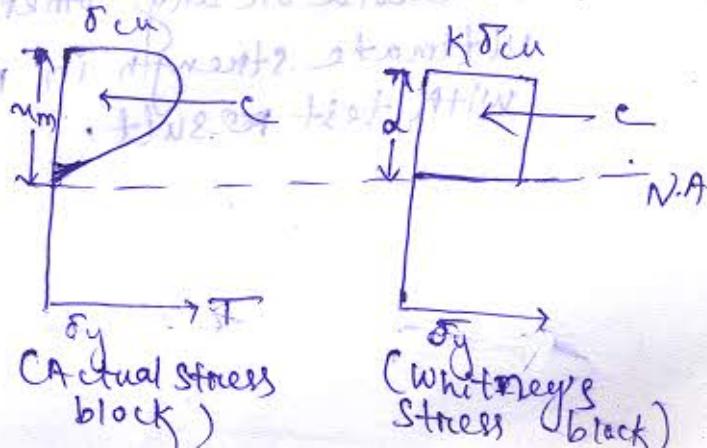
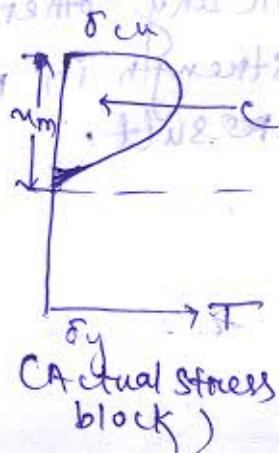
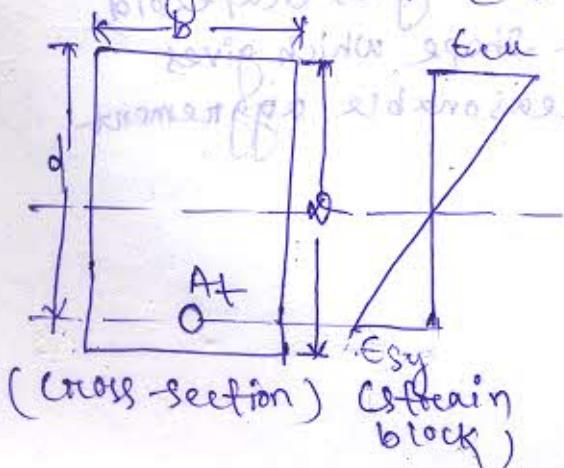
- In ultimate load method the working loads are increased by suitable factors to obtain ultimate loads. These factors are called load factors. The structure is then designed to resist the desired ultimate loads. This method takes into account the non linear stress strain behaviour of concrete.
- In working stress method factor of safety is defined as the ratio between yield stress & the working or permissible stress.
- $$F.O.S = \frac{\text{Yield Stress}}{\text{Working Stress}}$$

- Load factor is defined as the ratio of collapse or ultimate load to the working load.

## Assumptions of Whitney's theory:

- Ultimate strain in concrete is 0.3%.
- compressive stress at extreme edge of the section corresponds to the ultimate strain.
- Plane section before bending will remain plane after bending.

Whitney replaced the actual parabolic stress diagram by a rectangular stress diagram such that the C.G. of both the diagram lies at the same point & their areas are also equal. He found that the avg. stress of the rectangular stress diagram is equal to  $k\delta_{cu}$ .



Where,

- $a$  = depth of rectangular stress block.
- $= 0.537d$  in accordance with Whitney
- $= 0.43d$  in accordance with IS: 456-1964

$d_m$  = Depth of Neutral axis at failure.

$Z$  = lever arm.

$\delta_{cu}$  = ultimate compressive strength of concrete cube at 28 days

$k\delta_{cu}$  = Avg. stress

$= 0.85\delta_{cu}$  in accordance with Whitney

$= 0.55\delta_{cu}$  in accordance with IS 456-1964

$\delta'_{cu}$  = ultimate compressive strength of concrete cylinder at 28 days

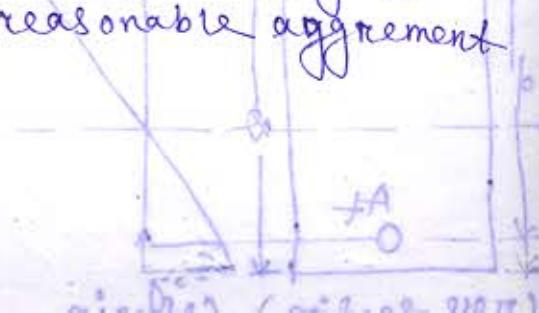
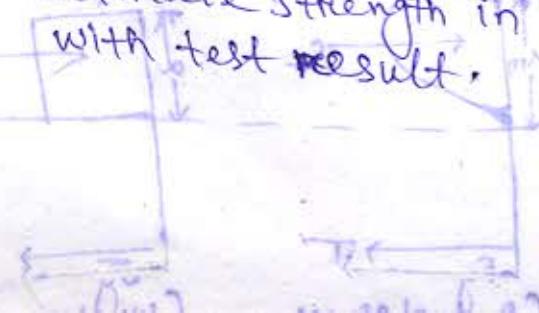
$\delta_y$  = yield stress in steel.

$\epsilon_{cu}$  = ultimate strain in concrete

$\epsilon_{sy}$  = yield strain in steel

Assumption in accordance with IS: 456-1964:

- A section which is plane before bending remains plane after bending.
- An ultimate strength Stress-Strain curve is not proportional & distribution of compressive stresses is non-linear in a section subjected to bending. The compressive stress diagram may be assumed as a rectangle, trapezoid, parabola or any other shape which gives ultimate strength in reasonable agreement with test result.



- (iii) Maximum fibre strength in concrete does not exceed  $0.68 \delta_{cu}$ . As in Whitney's theory, the actual stress diagram can be replaced by a rectangular stress-block whose height 'a' is taken  $0.43$  of the avg. stress is assumed to be  $0.55 \delta_{cu}$ .
- (iv) Tensile strength of concrete is ignored in sections subjected to bending.

The ultimate load - design method can be expressed as :

$$R > \lambda L$$

R - Resistance of the structural element.

L - Working loads on the structural element.

$\lambda$  - Load factor which is more than unity.

A major advantage of this method over the working stress method is that total safety factor of a structure thus found is nearer to its actual value. Moreover the structures designed by the ultimate load method generally require less reinforcement than those designed by working stress method.

### Drawbacks of Ultimate load Method:

Main drawbacks of ultimate load method are as follows :

- (i) Since load factor is used on the working loads. There is no way to account for different degree of uncertainties associated with variation in material stresses.
- (ii) There is complete disregard for control against excessive deflection.

### 3. Limit state method:

→ Limit state design has originated from ultimate or plastic design.

→ The object of design based on the limit state concept is to achieve an acceptable probability that a structure will not become unserviceable in its life time for the use for which it is intended, i.e. it will not reach a limit state.

IR. 8

→ A structure with appropriate degree of reliability should be able to withstand safety all loads that are liable to act on it throughout its life & it should also satisfy the serviceability requirement such as limitations on deflections & cracking.

→ It should be able to maintain the required structural integrity during & after accidents such as fires, explosions & local failure.

→ In other words all relevant limit states must be considered in design to ensure an adequate degree of safety & serviceability.

There are two methods of limit states.

1. Limit state of collapse

2. Limit state of Serviceability

## Limit State of Collapse :-

This state corresponds to the maximum load carrying capacity. Violation of collapse limit states implies failure in the sense that a clearly defined limit state of structural usefulness has been exceeded. However it does not mean a complete collapse.

This limit state may correspond to

- (a) Flexure
- (b) compression
- (c) Shear
- (d) Torsion

## Limit State of Serviceability :-

This state corresponds to development of excessive deformation & is used for checking members in which magnitude of deformations may limit the use of the structure or its components.

This limit state may correspond to

- (a) Deflection
- (b) cracking
- (c) vibration

The choice of degree of reliability should take into account the possible consequences of exceeding the limit state of collapse which may be classified according to

- (i) Risk to life is negligible & economic consequences small or negligible
- (ii) Risk to life exists & for economic consequences considered
- (iii) High risk to life great & economic consequences also great

- Elastic theory or working stress theory is generally applicable for serviceability limit state & fatigue.
- Plastic theory for ultimate limit states & stability analysis for overturning.
- In contrast to existing design methods limit state design applies to all kind of failure such as collapse, overturning & vibration & to all materials & type of construction. In short limit state design of building structures of all materials.

- The limit state concept of design of reinforced concrete structures takes into account the probabilistic & structural variation in material properties, loads & safety factors. Limit state of collapse can be expressed by the inequality.

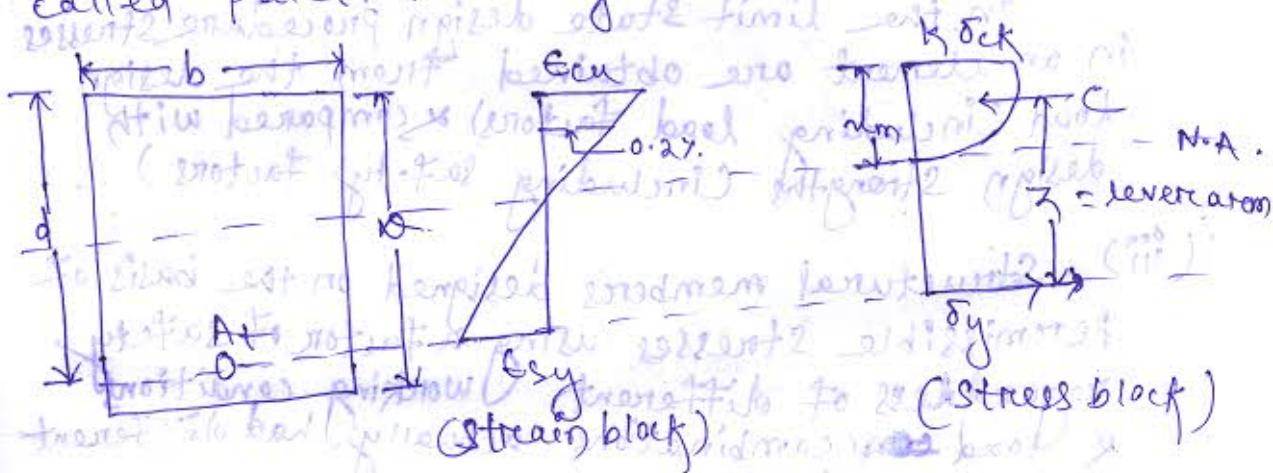
$$MR > \sum_{i=1}^n \lambda_i L_i$$

- Left hand side relates to the resistance or capacity of the structure. Right hand side relates characterised the load acting on it up to state limit. The summation sign represents the combination of load effect from different load source, for example dead load, live load, wind or earthquake load.

- The randomness in the resistance  $R$  of a structural element arises due to variation in material properties, workmanship & assumptions made in the theory underlying the definition of member strength.

- The safety factor  $\gamma$  which is always less than unity reflects the uncertainties associated with loads.
- The randomness in the evaluation of different loads ~~to~~ arises due to non-availability of sufficient & reliable data.
- The load factor  $\gamma_i$  which is normally greater than unity.

In limit state concept of design of reinforced concrete structures, the factors  $\mu$  &  $\lambda$  are called partial safety factors.



$\sigma_{ck}$  = characteristic strength of concrete.

$\gamma$  = safety factor.

The limit state of serviceability can be expressed by the inequality.

$$\frac{\delta}{L} \leq \frac{1}{\alpha}$$

$\delta$  = deflection

$L$  = length or height or span of the structural element

$\alpha$  = a constant non-dimensional number.

## Limit state method vs Working stress method :-

- (i) Working stress method is referred to as deterministic because it is presumed that loads, permissible stress & factor of safety are known accurately.
- Limit state method is referred to as Probabilistic because it is based on experience or on field data.

(ii) In working stress design method the stresses in an element are obtained from working load & compared with permissible stresses.

In the limit state design procedure stresses in an element are obtained from the design load (including load factors) & compared with design strengths (including safety factors).

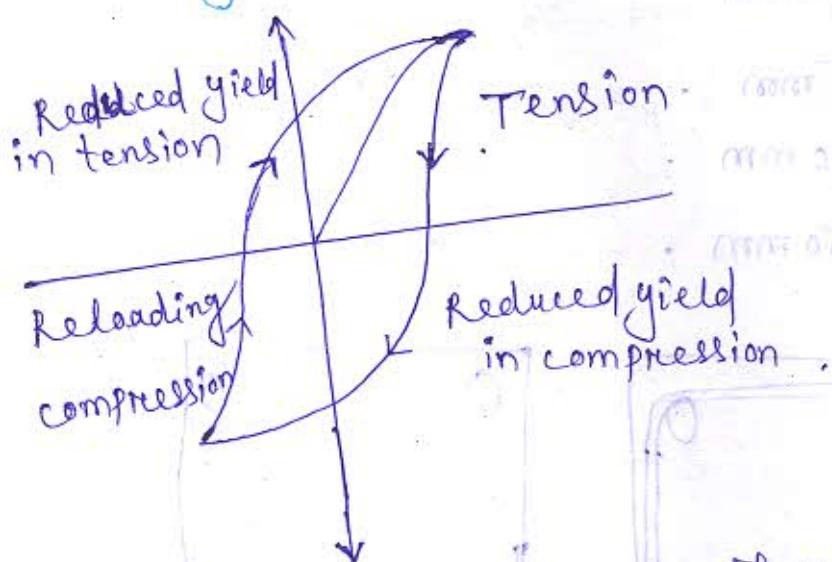
(iii) Structural members designed on the basis of permissible stresses using a factor of safety regardless of different working conditions & load combinations (actually had different safety margins).

The limit state method is based on physical parameters. The partial safety factors are based on statistical. It is a more scientific approach.

→ In limit state design method parameters are determined based on observations taken over period of time. These parameters will thus be influenced by chance or random effect not just at a single instant but throughout the entire period of time or the sequence of time that is being considered. Such process is known as stochastic process.

In a rough sense a stochastic process is a phenomena that varies to some degree unpredictably as time goes on. It is also referred to as a random process.

## Bauchinger Effect :-



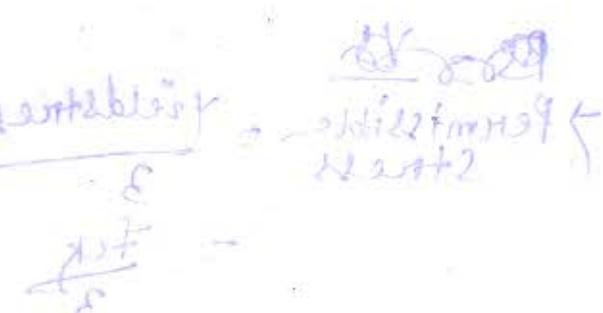
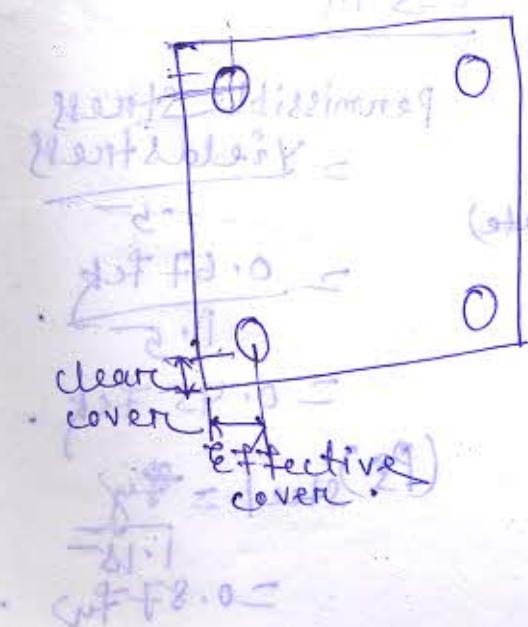
Stress-strain behaviour of mild steel in compression is identical to that of tension.

However if the steel is stressed into the elastic range in uniform tension, unloaded & then subjected to uniform compression that is reverse loading.

It is found that the stress-strain curve in compression becomes non-linear at a stress much lower than the initial yield

strength

## Nominal cover 'or' clear cover :- M 2 W



$$\frac{d}{8+1} = \text{height}$$

Nominal cover formulae under what

Slab - 20mm

Beam - 25mm

Column - 40mm

Footing - 50mm

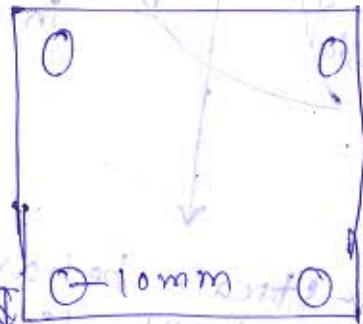


Fig-1

1. Effective cover =  $20 + \frac{10}{2} = 25\text{ mm}$

2. Effective cover =  $20 + 10 + \frac{10}{2} = 30\text{ mm}$

Factor of Safety = yield stress / Permissible stress.

Permissible stress = yield stress / F.O.S.

WSM

Permissible Stress

$$\rightarrow \text{Permissible Stress} = \frac{\text{Yield stress}}{3} \text{ (concrete)}$$
$$= \frac{f_{ck}}{3}$$

$\rightarrow$  Permissible stress in steel,

$$(P_s)_{\text{Steel}} = \frac{f_y}{1.8}$$

LSM

Permissible stress = Yield stress / 1.5

$$= \frac{0.67 f_{ck}}{1.5}$$
$$= 0.45 f_{ck}$$

$$(P_s)_{\text{Steel}} = \frac{f_y}{1.15}$$
$$= 0.87 f_y$$

## Modular Ratio:-

If it is the ratio of elasticity of steel to the elasticity of concrete.

It is represented by  $m$ .

$$m = \frac{E_s}{E_c} = \frac{2 \times 10^5}{5000 \text{ or } f_{ck}} \quad (\text{without considering creep})$$

If we consider the creep,

$$m = \frac{2 \times 10^5}{5000 \sqrt{f_{ck}}} (1 + \alpha)$$

$\alpha$  = creep co-efficient.

As per IS:456

$$m = \frac{280}{38 \text{ cbc}}$$

Date - 29/01/2019

The partial safety factor for steel is less than that of concrete because steel is manufactured in factories under proper quality control, whereas concrete is manufactured inside so the quality of concrete cannot be assured.

(Page - 68 - Table - 18)

Design Load = Load factor  $\times$  characteristics load.

~~Dead load + Imposed load~~  
(DL) ~~(IL)~~

~~Dead load~~

$$\frac{800.0 + 87F8.0}{2800.0}$$

$$\frac{\text{m.s. load}}{\text{m.s. capacity}}$$

$$\frac{800.0}{2800.0} + \frac{87F8.0}{2800.0} = 1 - \frac{b}{\text{m.s. capacity}}$$

Limit state of collapse

DL    LL    WL/EL

DL + LL/IL    1.5    1.5

DL + WL/EL    1.5

DL + IL + WL    1.2    1.2    1.2

Limit state of serviceability

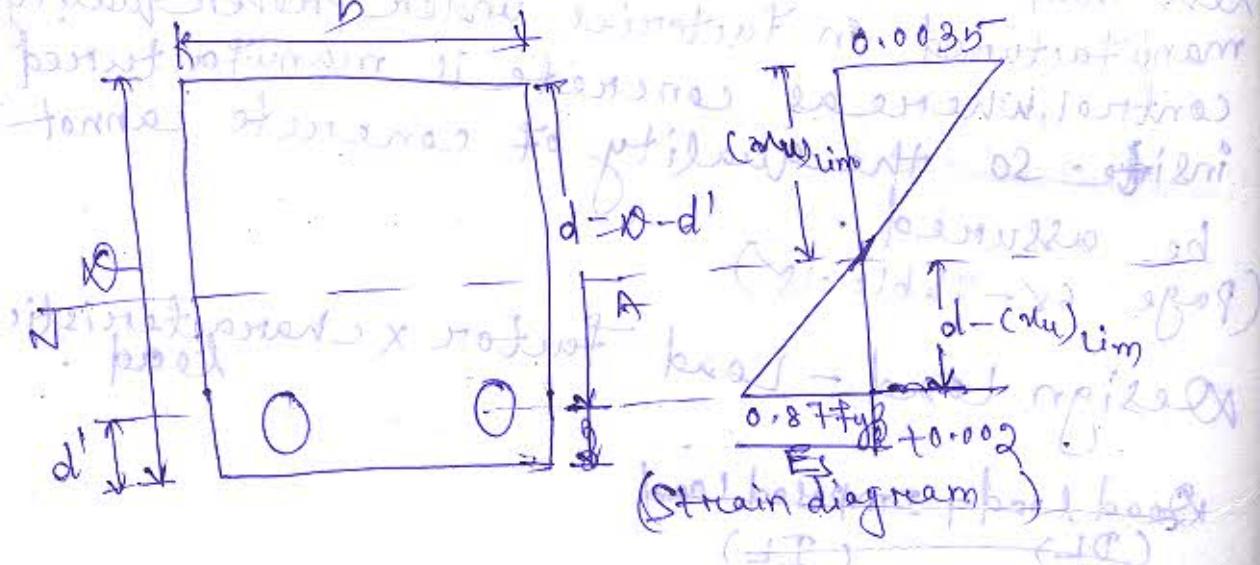
DL    LL    WL/EL

DL + IL    1.0    1.0

DL + WL/EL    1.0

DL + IL + WL    1.0

Analysis of singly reinforced beam



$$\frac{(d - \bar{e})u_{lim}}{(\bar{e}u)_{lim}} = \frac{0.87f_y + 0.002}{E_s \cdot 0.0035}$$

$$\Rightarrow \frac{d}{(u)_{lim}} - 1 = \frac{0.87f_y}{E_s \cdot 0.0035} + \frac{0.002}{0.0035}$$

$$\Rightarrow \frac{d}{(M_u)_{\text{lim}}} - 1 = \frac{0.87 f_y}{0.0035 \times 2 \times 10^5} + \frac{0.002}{0.0035}$$

$$= 1.24 \times 10^{-3} f_y + 0.571$$

$$\Rightarrow \frac{d}{(M_u)_{\text{lim}}} = 1.24 \times 10^{-3} f_y + 0.571 + 1$$

$$\Rightarrow \frac{(M_u)_{\text{lim}}}{d} = \frac{1}{1.24 \times 10^{-3} f_y + 1.571 + 1}$$

$$\Rightarrow \frac{(M_u)_{\text{lim}}}{d} = \frac{700}{1100 + 0.87 f_y}$$

$$\Rightarrow \left( \frac{(M_u)_{\text{lim}}}{d} \right)_{\text{Fe 250}} = \frac{700}{1100 + 0.87 \times 250}$$

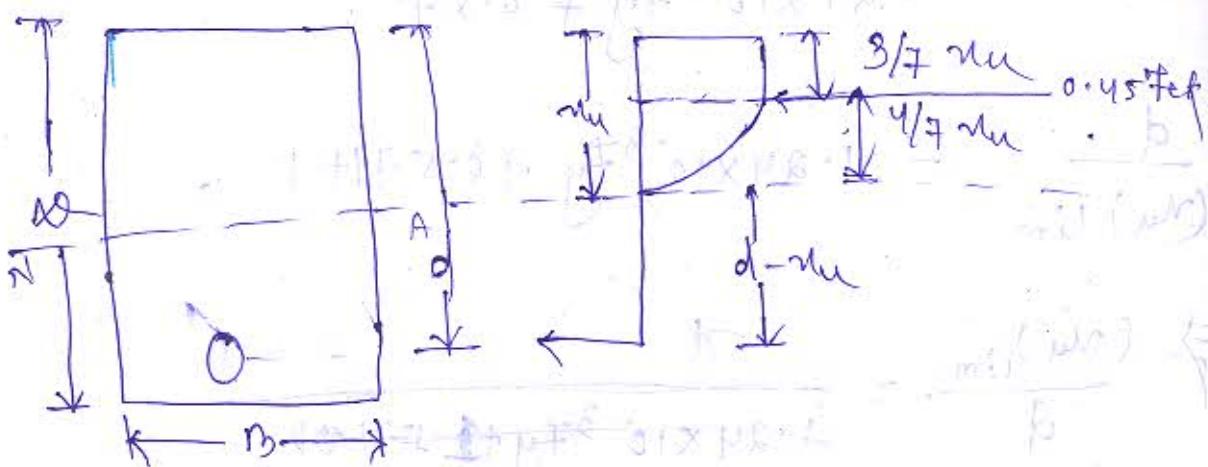
$$\text{Fe 415} \rightarrow \left( \frac{M_u}{d} \right)_{\text{lim}} = 0.48$$

$$\text{Fe 500} \rightarrow \left( \frac{M_u}{d} \right)_{\text{lim}} = 0.46$$

(minimum)  $M_u = 0.46 \times 700 = 322 \text{ Nmm}$

(maximum)  $M_u = 0.46 \times 1100 = 506 \text{ Nmm}$

## Analysis of Stress diagram



$$C_1 = 0.45 f_{ck} \times A,$$

$$= 0.45 f_{ck} \times B \times \frac{3}{7} m.$$

$$C_2 = 0.45 f_{ck} \times \frac{2}{3} \times \frac{4}{7} m \times B.$$

$$C = C_1 + C_2$$

$$= 0.45 f_{ck} \times \frac{3}{7} m \times B + 0.45 f_{ck} \times \frac{4}{7} m \times \frac{2}{3} \times B$$

$$= 0.45 f_{ck} \times m \times B \times \left( \frac{3}{7} + \frac{4}{7} \times \frac{2}{3} \right)$$

$$= 0.45 f_{ck} \times m \times B \times 0.869$$

$$= 0.36 f_{ck} m \times B.$$

$$\text{Force of steel} = 0.87 f_y \times A_{st}$$

$$0.36 f_{ck} m \times B = 0.87 f_y A_{st}.$$

$$\text{Moment} = 0.87 f_y A_{st} (d - 0.42m) \text{ (Tensile)}$$

$$= 0.36 f_{ck} m \times B (d - 0.42m) \text{ (Compressive)}$$

- When  $\mu_e < (\mu_e)_{\text{lim}}$ , the beam is under-reinforced
- When  $\mu_e = (\mu_e)_{\text{lim}}$ , the beam is a balanced section.
- When  $\mu_e > (\mu_e)_{\text{lim}}$ , the beam is over-reinforced.

Restrict the value of  $\mu_e$  to  $(\mu_e)_{\text{lim}}$ .  
or redesign the beam.

$$(\text{M.O.R})_{\text{lim}} = \begin{cases} 0.148 f_{ck} bd^2 & (\text{F}_200) \\ 0.138 f_{ck} bd^2 & (\text{F}_415) \\ 0.133 f_{ck} bd^2 & (\text{F}_500) \end{cases}$$

Date - 01/02/2019

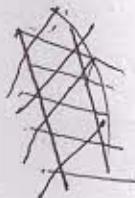
### Formula

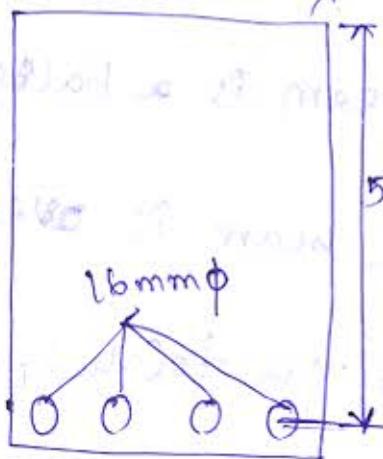
$$1. \frac{A_{st}}{bd} = \frac{f_{st}}{f_y} \times 100 \quad (\text{Page} - 89)$$

$$2. A_{st} = 0.15 \frac{f_{ck}}{f_y} \left[ 1 - \sqrt{1 - \frac{4.6 M_e}{f_{ck} b d^2}} \right] \times b d$$

### Problem :-

A rectangular beam 230 mm wide 520 mm effective depth is reinforced with 4 nos. of 16 mm dia bars. Find out the depth of neutral axis & specify the type of concrete & H.S.D reinforcement of beam. The materials are M20 grade concrete & H.S.D reinforcement of Fe415. Also find out the depth of neutral axis if the reinforcement is increased to 4 no. of 20 mm dia bars.





$$b = 230 \text{ mm}$$

$$d = 520 \text{ mm}$$

$$\begin{aligned} A_{st} &\geq \frac{\pi}{4} \times d^2 \\ &= \frac{\pi}{4} \times (16)^2 \times 4 \\ &= 804.24 \approx 804 \text{ mm}^2 \end{aligned}$$

$$f_{ck} = 20 \text{ N/mm}^2 = 20 \text{ MPa}$$

$$f_y = 415 \text{ N/mm}^2$$

$$b \times c = T - 1 \quad k = 1 \quad \frac{T}{k} = 8.0 = kA$$

$$\Rightarrow 0.36 f_{ck} b = 0.87 f_y A_{st}$$

$$\Rightarrow \text{Area } b \times c = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$\text{Area } b \times c = \frac{0.87 \times 415 \times 804}{0.36 \times 20 \times 230} = 175.3 \text{ mm}^2$$

$$\text{Area } b \times c = 175.3 \text{ mm}^2$$

$$\begin{aligned} \Rightarrow M_u &= 0.48 \times 520 \\ &= 249.6 \text{ mm} \end{aligned}$$

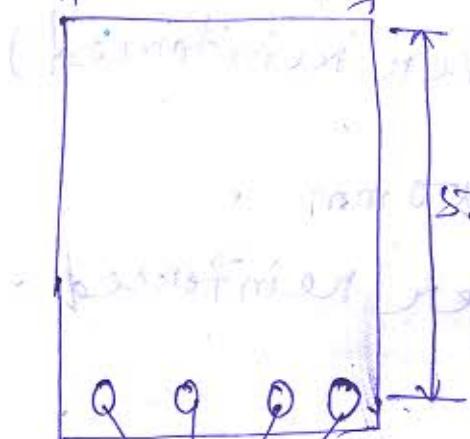
$\mu_e < (\mu_u)_{lim}$  (under-reinforced).

$$m = 175 \text{ mm}.$$

So, the beam is under-reinforced.

$\phi 20 \text{ mm} \cdot \text{ min. PPS} =$   
Astros

230 mm



if rig = 4 no. 20 mm  $\phi$

$$b = 230 \text{ mm}$$

$$d = 2520 \text{ mm}$$

$$\text{1. } A_{st} = \frac{\pi}{4} \times 20^2 \times 4$$

$$= 1256.64$$

$$\approx 1257 \text{ mm}^2$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$0.87 c = T$$

$$\Rightarrow 0.36 f_{ck} m b = 0.87 f_y A_{st}$$

$$\Rightarrow \mu_e = \frac{0.87 \times 415 \times 230 \times 1257}{0.36 \times 20 \times 230}$$

$$= 274.1 \text{ mm}.$$

$$\frac{(M_u)_{\text{lim}}}{d} = 0.48$$

$$\Rightarrow (M_u)_{\text{lim}} = 0.48 \times 1520 = 249.6 \text{ mm}$$

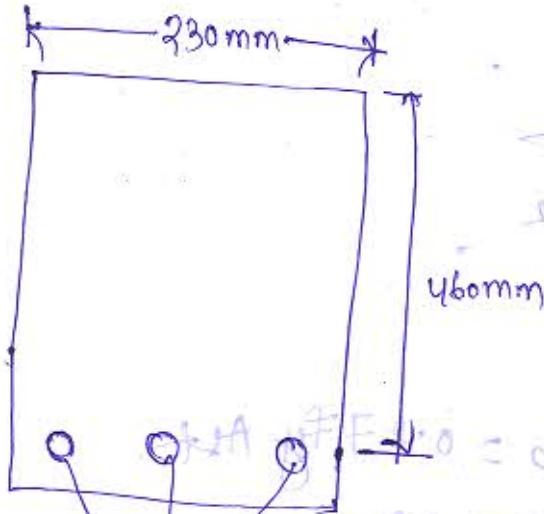
$M_u > (M_u)_{\text{lim}}$  (Cover reinforced)

$$M_u = 249.6 \text{ mm} \leq 250 \text{ mm}$$

So, the beam is over reinforced.

Q: A singly reinforced rectangular beam width of 230 mm & effective depth is 460 mm reinforced with 3 nos. of 20 mm dia bars. Find out the factored moment of resistance of the section, the material are M20 grade concrete & Fe415 steel.

Sol:



$$F_{ck} = 21 \text{ N/mm}^2 \times F_{8.0} = 167.2 \text{ N/mm}^2$$

3 nos. 20mm dia

$$F_y = 415 \text{ N/mm}^2$$

$$b = 230 \text{ mm}$$

$$d = 460 \text{ mm}$$

$$\begin{aligned} A_{st} &= 3 \times \frac{\pi}{4} \times 20^2 \\ &= 943 \text{ mm}^2 \end{aligned}$$

$$f_{ck} = 20 \text{ MPa}$$

$$f_y = 415 \text{ MPa}$$

$$c = T$$

$$\Rightarrow 0.36 f_{ck} \gamma_{mb} = 0.87 f_y A_{st}$$

$$\begin{aligned} \Rightarrow \gamma_m &= \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} \\ &= \frac{0.87 \times 415 \times 943}{0.36 \times 20 \times 230} \\ &= 206 \text{ mm} \end{aligned}$$

$$\frac{(\gamma_m)_{lim}}{d} = 0.48$$

$$\begin{aligned} \Rightarrow (\gamma_m)_{lim} &= 0.48 \times d \\ &= 0.48 \times 460 \\ &= 221 \text{ mm} \end{aligned}$$

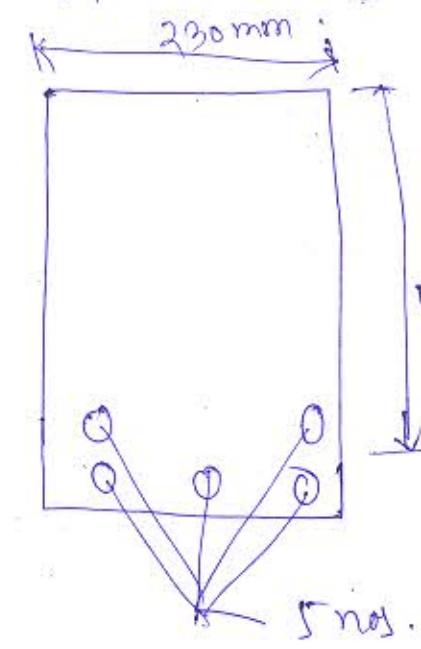
$$\gamma_m < (\gamma_m)_{lim}$$

So the section is under reinforced

$$\gamma_m = 206 \text{ mm}$$

$$\begin{aligned} M.O.R &= 0.87 f_y A_{st} (d - 0.42 \gamma_m) \\ &= 0.87 \times 415 \times \frac{943}{460 - 0.42 \times 206} \\ &= 132570.34 \text{ Nmm} \\ &= 127158791.6 \text{ Nmm} \\ &\approx 125.01 \text{ kNm} \\ &\approx 127.15 \text{ kNm} \end{aligned}$$

No. of bars = 5



$$b = 230 \text{ mm}$$

$$d = 460 \text{ mm}$$

$$f_{ck} = 20 \text{ MPa}$$

$$f_y = 415 \text{ MPa}$$

$$A_{st} = \frac{\pi}{4} \times 20^2 \times 5$$

$$= 1570.79 \text{ mm}^2$$

$$= 1571 \text{ mm}^2$$

compression = Tension

$$\Rightarrow 0.36 f_{ck} m_u b = 0.87 f_y A_{st}$$

$$\Rightarrow m_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 415 \times 1571}{0.36 \times 20 \times 230}$$

$$= 342.52 \text{ mm}$$

$$= 343 \text{ mm}$$

$$\frac{(m_u)_{lim}}{d} = 0.48$$

$$\Rightarrow (m_u)_{lim} = 0.48 \times d.$$

$$= 0.48 \times 460$$

$$= 220.8 \text{ mm}$$

$$\approx 221 \text{ mm}$$

M.O.R.  $m > (m_u)_{lim}$  so the section is overreinforced,

$$m_u = 221 \text{ mm}$$

$$\text{M.O.R.} = 0.36 f_{ck} m_u b (d - 0.42 m_u)$$

for compression

$$= 0.36 \times 20 \times 221 \times 230 (460 - 0.42 \times 221)$$

$$= 134379067.7 \text{ Nmm}$$

$$= 134.37 \text{ kNm}$$

$$\text{M.O.R.} = 0.87 f_y A_s t (d - 0.42 m_u)$$

for tension

$$= 0.87 \times 415 \times 1571 \times (460 - 0.42 \times 221)$$

$$= 208268002.6 \text{ Nmm}$$

$$= 208.26 \text{ kNm}$$

Q: A singly reinforced beam is subjected to a bending moment of 36 kNm at working load. The width of the beam is 200 mm. Find the depth & steel area for balanced design. The material is M<sub>20</sub> concrete & mildsteel.

$$\frac{M \times R / 20}{508 \times 17} = \pi f_c$$

$$f_{ck} = 20 \text{ MPa}$$

$$f_y = 250 \text{ MPa}$$

$$b = 200 \text{ mm}$$

$$\text{Working moment} = 36 \text{ kNm}$$

$$\text{Factored moment} = 36$$

$$\text{Design moment} = 36 \times 1.5 =$$

$$= 54 \text{ kNm}$$

$$= 54 \times 10^6 \text{ Nmm}$$

$$(Mu)_{\text{lim}} = 0.148 + f_{ck} bd^2$$

$$\Rightarrow 54 \times 10^6 = 0.148 \times 20 \times 200 \times d^2$$

$$\Rightarrow d = \sqrt{\frac{54 \times 10^6}{0.148 \times 20 \times 200}}$$

$$= 302.02$$

$$\approx 302 \text{ mm}$$

$$A_{st} = 0.5 \frac{f_{ck}}{f_y} \left[ 1 - \sqrt{1 - \frac{4.6 Mu}{f_{ck} bd^2}} \right] \times b d$$

$$= 0.5 \times \frac{20}{250}$$

$$\left[ 1 - \sqrt{1 - \frac{4.6 \times 54 \times 10^6}{20 \times 200 \times (302)^2}} \right]$$

$$= 1051 \text{ mm}^2$$

$$\Rightarrow n = \frac{1051 \times 4}{\pi \times 20^2}$$

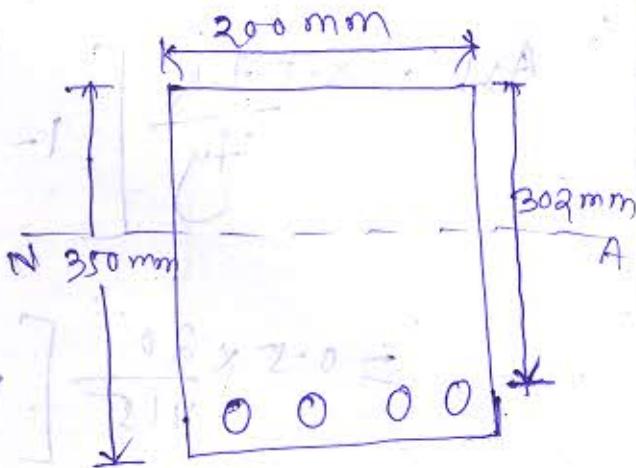
$$= 3.34 \approx 4 \text{ nos.}$$

Date - 04/02/2019

Ans. of 20mm  $\phi$  bar

Overall depth,

$$\begin{aligned} D &= 302 + \frac{20}{2} + 25 \\ &= 337 \text{ mm} \\ &\stackrel{?}{=} 350 \text{ mm} \end{aligned}$$



Q: Design a singly reinforced rectangular beam for an applied factored moment of 120 kNm. Assume the width of the section is 230 mm. The materials are M20 grade concrete & Fe415 Steel.

Soln

Factored moment,  $M_f = 120 \text{ kNm} \approx 120 \times 10^6 \text{ Nmm}$ .

width,  $b = 230 \text{ mm}$

$f_{ck} = 20 \text{ N/mm}^2$

$f_{y} = 415 \text{ N/mm}^2$

$$(M_u)_{\text{lim}} = 0.158 f_{ck} b d^2$$

$$\Rightarrow 120 \times 10^6 \approx 0.158 \times 20 \times 230 \times d^2$$

$$\Rightarrow d = \sqrt{\frac{120 \times 10^6}{0.158 \times 20 \times 230}}$$

$$= \cancel{419.83} 434.78 \text{ mm}$$

$\stackrel{?}{=} 440 \text{ mm}$

$$\frac{L \times N \times 25P}{0.85} = r$$

$$F.P. \cdot S =$$

$$A_{st} = \frac{0.5 f_y k}{f_y} \left[ 1 - \sqrt{1 - \frac{4.6 M_u}{f_y k b d^2}} \right] x b d$$

$$= 0.5 \times \frac{20}{415} \left[ 1 - \sqrt{1 - \frac{4.6 \times 120 \times 10^6}{20 \times 230 \times 440^2}} \right] x 230 \times 440$$

$$= 935.00 \text{ mm}^2$$

Let us provide 16 mm dia bars

$$n \times \frac{\pi}{4} \times 16^2 = 935$$

$$\Rightarrow n = \frac{935 \times 4}{\pi \times 16^2}$$

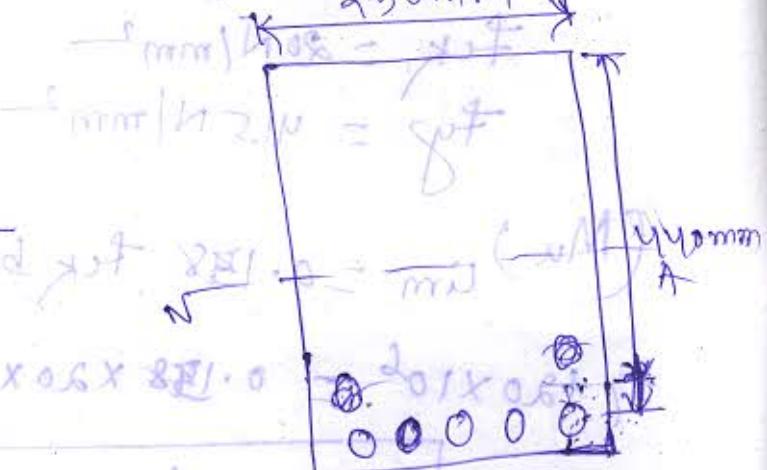
$\approx 5$  nos.

Provide 5 nos. of 16 mm dia bars

Overall depth,

$$D = 440 + \frac{16}{2} + 25 = 473$$

$$\approx 480 \text{ mm}$$



Let us provide 20 mm dia bars,

$$n \times \frac{\pi}{4} \times 20^2 = 935$$

$$\Rightarrow n = \frac{935 \times 4}{\pi \times 20^2}$$

$$\approx 2.97$$

$$\approx 3 \text{ nos.}$$

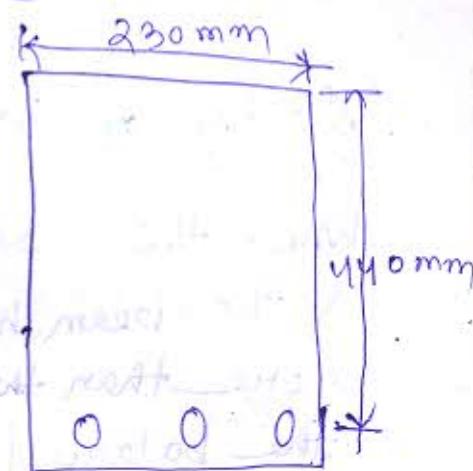
Provide 3 nos. of 20mm  $\phi$  bars

Overall depth

$$= 440 + \frac{20}{2} + 25$$

$$= 475$$

$$\approx 480 \text{ mm}$$



### Design of Doubly reinforced beam



$$(1 - \frac{b}{\mu r}) 2800.0 = 4237$$

$$\frac{b - \mu r}{\mu r} = \frac{183}{7000.0}$$

$$(\frac{b}{\mu r} - 1) 2800.0 = 327$$

$$T = 3$$

$$f_{ck} f_{st} = 0.7 f_{ck} + 0.7 f_{st} 210.0 = 280.0$$

$$g_{sf} f_{st} 8.0 = f_{st} = 28$$